

Key

Half-life

Half-life of a reaction, $t_{1/2}$: the time required for a reactant's [] to drop to $\frac{1}{2}$ of its original value.

Radioactive decay is a first order reaction.

$$[A]_{t_{1/2}} = \frac{1}{2} [A]_0$$

From the basic equation $\ln [A]_t = \ln [A]_0 - kt$

Put $\frac{1}{2} [A]_0$ here \uparrow and $t_{1/2}$ here, and we get $t = .693/k$

For 1st order reactions...

- $t_{1/2}$ is independent of initial concentration
- the concentration of reactants is cut in half... every half-life

$$N = N_0(1/2)^n$$

N = amount remaining

N_0 = original amount

n = # of half-life(s)

1. The half-life of cesium-137 is 3.2 years. If the initial mass of a sample of cesium-137 is 1.0 kg, how much will remain after 151 years?

$$\ln \frac{[A]_0}{[A]} = kt = 32.701 \quad \frac{[A]_0}{[A]} = 1.59 \times 10^{14} \quad k = .693/3.2 = 0.2166$$

$$\frac{1}{1.59 \times 10^{14}} = 6.29 \times 10^{-15} \text{ kg}$$

2. Given that the half-life of carbon-14 is 5730 years, consider a sample of fossilized wood that, when alive, would have contained 24 g of carbon-14. It now contains 1.5 g of carbon-14. How old is the sample?

$$k = \frac{.693}{5730} = 1.21 \times 10^{-4} \text{ yr}^{-1} \quad t = \ln \left(\frac{24}{1.5} \right) / k = 2.773$$

$$t = \frac{2.773}{1.21 \times 10^{-4}} = 22914 \text{ years}$$

3. A 64-g sample of germanium-66 is left undisturbed for 1.5 hours. At the end of that period, only 2.0 g remain. What is the half-life of this material?

$$\ln \left(\frac{64}{2} \right) = kt = 3.466 = k(1.5) \quad k = 2.310 \quad t_{1/2} = \frac{.693}{2.310} = 0.30 \text{ hrs} = 18 \text{ min}$$

4. With a half-life of 28.2 years, how long will it take for 1 g of strontium-90 to decay to 125 mg?

$$\ln \left(\frac{1}{.125} \right) = (.025)(t) \quad t = 83.18 \text{ yrs} \quad k = \frac{.693}{28.2} = 0.025$$

5. Cobalt-60 has a half-life of 5.3 years. If a pellet that has been in storage for 26.5 years contains 14.5 g of cobalt-60, how much of this radioisotope was present when the pellet was put into storage?

$$\ln \frac{[A]_0}{14.5} = kt \quad k = \frac{.693}{5.3} = 0.131$$

$$\ln [A]_0 = \ln [A]_t + kt = 6.146 \quad [A]_0 = 466.85 \text{ g}$$

6. A 1000 kg block of phosphorous-32, which has a half-life of 14.3 days, is stored for 100.1 days. At the end of this period, how much phosphorous-32 remains?

$$\ln [A]_t = -kt + \ln [A]_0 \quad k = \frac{.693}{14.3} = 0.048$$

$$= -(0.048)(100.1) + \ln(1000)$$

$$= 2.103$$

$$[A]_t = 8.19 \text{ kg}$$

7. A sample of air from a basement is collected to test for the presence of radon-222, which has a half-life of 3.8 days. However, delays prevent the sample from being tested until 7.6 days have passed. Measurements indicate the presence of 6.5 μg of radon-222. How much radon-222 was present in the sample when it was initially collected?

$$\ln [A]_0 = \ln [A]_t + kt$$

$$= \ln(6.5) + (0.182)(7.6) = 3.255$$

$$[A]_0 = 25.92 \mu\text{g}$$

$$k = \frac{.693}{3.8} = 0.182$$

8. A 0.500 M solution of iodine-131, which has a half-life of 8.0 days, is prepared. After 40.0 days, how much iodine will remain in 1.0 L of solution? Express the results in moles.

$$\ln [A]_t = -kt + \ln [A]_0 = -(0.087)(40) + \ln(0.5)$$

$$= -4.173$$

$$[A]_t = 0.015 \text{ M} = 0.015 \text{ moles in 1 L}$$

$$k = \frac{.693}{8} = 0.087$$

- 9) The half-life of sodium-25 is 1.0 minute. Starting with 1 kg of this isotope, how much will remain after an hour?

$$\ln [A]_t = -(0.693)(60) + \ln(1)$$

$$= -41.58$$

$$[A]_t = 8.75 \times 10^{-19} \text{ kg} = 8.75 \times 10^{-16} \text{ g}$$

$$k = \frac{.693}{1} = .693$$

10. What is the half-life of polonium-214 if, after 820 seconds, a 1.0 g sample decays to 31.25 mg?

$$\ln\left(\frac{1}{.03125}\right) = kt = (k)(820)$$

$$= 3.466$$

$$k = 0.00423$$

$$t_{1/2} = \frac{.693}{.00423}$$

$$= 163.83 \text{ s}$$

11. $2\text{N}_2\text{O}_5(\text{g}) \longrightarrow 4\text{NO}_2(\text{g}) + \text{O}_2(\text{g})$

Dinitrogen pentoxide decomposes to nitrogen dioxide and oxygen gas via a first-order reaction with a rate constant of $5.1 \times 10^{-4} \text{ s}^{-1}$ at 45°C . If the concentration of N_2O_5 is 0.30M, what is the concentration after 180 sec?

$$\ln [A]_t = -kt + \ln [A]_0 = -(5.1 \times 10^{-4})(180) + \ln(0.30) = -1.30$$

$$[A]_t = 0.27 \text{ M}$$

How long will it take (in sec) for the concentration of N_2O_5 to decrease in half (from 0.30M to 0.15M)?

$$t = \frac{.693}{5.1 \times 10^{-4}} = 1359 \text{ s}$$

How long will it take to convert 75% of the starting material?

$$t = \frac{\ln(4)}{5.1 \times 10^{-4}} = 2718 \text{ s}$$

1. 0.0313 kg
6. 7.81 g

2. 23,000 years
7. 26 μg

3. 18 minutes
8. 0.016 mol

4. 90 years
9. $9 \times 10^{-7} \text{ g}$

5. 460 g
10. 160 s

11. 0.27 M, 1359 s, 2718 s